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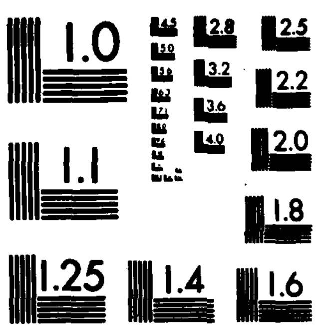

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ON SELECTING THE BEST AMONG GOOD POPULATIONS BASED ON A TWO-STAGE PROCEDURE: A BAYESIAN APPROACH WITH APPLICATIONS*

by

Shanti S. Gupta** and Joong K. Sohn
Purdue University

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Abstract

Several procedures have been studied to select the best among a set of new treatments (populations) which are better than a standard (or control) using two-stage procedures for the case of normal populations. One such procedure is to select the best based on the confidence intervals with a specified fixed width 2d after eliminating those populations which are worse than the standard based on the expected posterior losses. Several papers deal with this kind of problem but none of them is based on the so-called $100(1-2\alpha)\%$ Highest Posterior Density (HPD) credible regions, which are conceptually equivalent to the confidence intervals, with a fixed width 2d. After retaining good populations based on the expected posterior losses, we set up a stopping rule N_i for constructing the HPD credible region for each selected population, which is asymptotically efficient and consistent. Thereafter, we develop several different decision criteria based on the whole samples or the HPD credible regions. For applications, we use a noninformative prior for the unknown means θ and unknown variances σ^2 of normal populations, which might lead to robustness: Here we use $0 - k_i$ losses at Stage 1 and a stopping rule N_i which provides a $100(1-2\alpha)\%$ HPD credible region for each selected population with a fixed width 2d to decide on the choice of the best population based on the overall sample means at Stage 2.

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1. Introduction

Since the early work of Bechhofer, Dunnett and Sobel (1954) on the two-sample (two-stage) problem for selecting the normal population associated with the largest unknown mean from $k(\geq 2)$ normal populations, several different two-stage procedures have been studied for the following three cases: (i) known variances, (ii) common unknown variances and (iii) unknown and unequal variances.

Among these different procedures, there are mainly two types: (a) elimination type rules which select a subset by eliminating some non-best populations at the first stage and take additional samples according to the sampling scheme based on some design criteria to decide on the choice of the best at the second stage, and (b) nonelimination type procedures for which one decides on sample sizes at the first stage and then takes additional samples on all populations so as to decide on the selection of the best. Most of these procedures use the so-called indifference zone approach introduced by Bechhofer (1954), and especially for the elimination type procedures, at the first stage subset selection procedures are used: this approach was introduced by Gupta (1956).

For the elimination type procedures, Alam (1970) studied the known variances case. Tamhane and Bechhofer (1977, 1979), using a minimax criterion, also studied the known variances case. Gupta and Kim (1982) and Tamhane (1975) have considered the common unknown variances case.

For the nonelimination type rules, Bechhofer, Dunnett and Sobel (1954) have studied the common unknown variances case and Dudewicz and Dalal (1975), Rinott (1978), Bofinger (1979) and Mukhopadhyay (1979) have considered the problem for the unknown and unequal variances case. Recently Gupta and Miescke (1981, 1982), among others, have studied the problem under a decision-theoretic Bayesian framework.

In this paper, we propose an elimination type procedure with Bayesian setting, which retains good populations based on the expected posterior loss. We use certain loss functions and prior distributions. We also use a stopping rule to construct the $100(1-2\alpha)\%$ Highest Posterior Density (HPD) credible region, which is equivalent to a $100(1-2\alpha)\%$ confidence interval conceptually, with a fixed width 2d. Then we decide on the selection of the best based on some criteria.

For an application of this procedure, we use a $0 - k_i$ type loss function and a noninformative prior for unknown parameters and select the best based on the overall sample means.

2. General Framework for the Proposed Procedure $R(\alpha,d)$.

Let π_i, $i = 1,2,\ldots,k$, be k normal populations with unknown mean θ_i and unknown variances σ_i^2 $(0 < \sigma_i^2 < \infty)$. Also let X_i be the (observable) characteristic associated with π_i and let its probability density function be $f(x_i|\theta_i, \sigma_i^2)$. For $i = 1,2,\ldots,k$, let $\underset{\sim}{x}_i = (x_{i1},\ldots,x_{in_i})$ be n_i realizations of the random variable X_i. Let $\tau(\theta_i, \sigma_i^2)$ be a prior distribution of (θ_i,σ_i^2) which is absolutely continuous. Then if $\tau(\theta_i,\sigma_i^2|\underset{\sim}{x}_i)$ is the posterior distribution of (θ_i,σ_i^2), then by definition,

$$(2.1)\qquad \tau(\theta_i,\sigma_i^2|\underset{\sim}{x}_i) = \frac{\tau(\theta_i,\sigma_i^2)\prod_{j=1}^{n_i} f(x_{ij}|\theta_i,\sigma_i^2)}{m(\underset{\sim}{x}_i)},$$

where

$$(2.2)\qquad m(\underset{\sim}{x}_i) = \iint \tau(\theta_i,\sigma_i^2)\prod_{j=1}^{n_i} f(x_{ij}|\theta_i,\sigma_i^2)\,d\theta_i\,d\sigma_i^2.$$

Also the marginal posterior distributions of θ_i and σ_i^2 can be obtained by

$$(2.3) \qquad \tau_1(\theta_i|\underline{x}_i) = \int \tau(\theta_i,\sigma_i^2|\underline{x}_i)d\sigma_i^2,$$

and

$$(2.4) \qquad \tau_2(\sigma_i^2|\underline{x}_i) = \int \tau(\theta_i,\sigma_i^2|\underline{x}_i)d\theta_i.$$

π_i is said to be good if $\theta_i \in (c,\infty)$, where c is a constant specified 'a priori' by the experimenter. Then our loss structure is as follows:

$$(2.5) \qquad L(\theta_i,a_p) = \begin{cases} 0 & \text{if } \theta_i \in \Theta_p,\ p = 0,1 \\ \ell_p(c,\theta_i) & \text{if } \theta_i \in \Theta-\Theta_p, \end{cases}$$

where $\Theta = \mathbb{R}^1$, $\Theta_0 = (c,\infty)$, and where the action space $\mathcal{A} = \{a_0,a_1\}$. Here the action a_0 accepts π_i as a good population and the action a_1 rejects π_i as a non-best (non-contending) population.

<u>Definition</u> (see Berger (1980)). The $100(1-2\alpha)\%$ HPD credible region for θ_i is the subset $C_i(1-2\alpha)$ of Θ of the form

$$(2.6) \qquad C_i(1-2\alpha) = \{\theta_i \in \Theta;\ \tau_1(\theta_i|\underline{x}_i) \geq k(2\alpha)\},$$

where $k(2\alpha)$ is the largest constant such that

$$(2.7) \qquad Pr(C_i(1-2\alpha)|\underline{X}_i = \underline{x}_i) \geq 1-2\alpha.$$

<u>Remark</u>: If $\tau_1(\theta_i|\underline{x}_i)$ is not unimodal, the credible region $C_i(1-2\alpha)$ consists of several disjoint intervals. To avoid this kind of complexity, we assume here that $\tau_1(\theta_i|\underline{x}_i)$ is (strongly) unimodal. Let $C_i(1-2\alpha) \equiv (a_i,b_i) \subseteq \mathbb{R}^1$. Then $C_i(1-2\alpha)$ can be constructed by the following equations.

$$(2.8) \qquad \tau_1(a_i|\underline{x}_i) = \tau_1(b_i|\underline{x}_i),$$

$$(2.9) \qquad \int_{a_i}^{b_i} \tau_1(\theta_i|\underline{x}_i)d\theta_i \geq 1-2\alpha,$$

Hence, now we can set up a stopping rule N_i for each π_i selected at Stage 1 as follows:

At Stage 2, take $N_i - n_0$ additional observations from each selected π_i such that

$$N_i = \inf\{n;\ n \geq n_0 \text{ and } n \geq \eta(\underset{\sim}{x}_i, \alpha, d)\}, \tag{2.14}$$

where $\eta(\underset{\sim}{x}_i, \alpha, d)$ is the solution of the equation (2.13) and $\eta(\alpha, d)$ which is used at Stage 1 is decided based on the known proportion of $\lim\limits_{n_i \to \infty} \eta(\underset{\sim}{x}_i, \alpha, d)$.

Then two possible orders are

$$a_{[1]} \leq a_{[2]} \leq \cdots \leq a_{[s]}, \tag{2.15}$$

and

$$b_{[1]} \leq b_{[2]} \leq \cdots \leq b_{[s]}, \tag{2.16}$$

where s is the size of the subset S selected at Stage 1. Hence we can denote

$$C_{(1)}(1-2\alpha) \leq C_{(2)}(1-2\alpha) \leq \cdots \leq C_{(s)}(1-2\alpha) \tag{2.17}$$

or

$$\pi_{(1)} \leq \pi_{(2)} \leq \cdots \leq \pi_{(s)} \tag{2.18}$$

corresponding to the order (2.15) or (2.16). Then we can select populations based on the following two decisions.

Decision A. If we define the population $\pi_{(s)}$ associated with the credible region $C_{(s)}(1-2\alpha)$ the best, then select $\pi_{(i)}, \pi_{(i+1)}, \ldots, \pi_{(s)}$ corresponding to $C_{(i)}(1-2\alpha)$, $C_{(i+1)}(1-2\alpha), \ldots, C_{(s)}(1-2\alpha)$, where i is the first j such that

$$b_{[j]} \geq a_{[s]} + d_0, \tag{2.19}$$

where d_0 is defined by a suitable condition on the minimum probability of a correct selection (PCS).

Decision B. If the population π_i with the largest unknown mean among good ones is defined as the best, then the selection procedure as follows:

$$\text{Select } \pi_i \text{ corresponding to } \bar{X}_i = \max_{\pi_j \in S} \frac{1}{N_j} \sum_{\ell=1}^{N_j} X_{j\ell}.$$

Remark. The minimum PCS may be invoked depending upon the type of the decision. Also the minimum PCS can vary depending on the type of the problem.

3. An Application of the Procedure $R(\alpha,d)$.

Let π_i, $i = 1,2,\ldots,k$ be k normal populations with unknown mean θ_i and unknown variance σ_i^2 $(0 < \sigma_i^2 < \infty)$ and let X_{ij}, $j = 1,2,\ldots,n_0$: $i = 1,2,\ldots,k$ be n_0 independent samples from a partition π_i, where n_0 is defined later. We define a population π_i to be good if $\theta_i \in (c,\infty)$, where c is a constant a priori specified by the experimenter. Our goal is to select the population associated with the largest θ_i among good populations. Let our loss be as follows:

$$L(\theta_i,a_p) = \begin{cases} 0 & \text{if } \theta_i \in \Theta_p, \quad p = 0,1, \\ k_p & \text{if } \theta_i \in \Theta-\Theta_p, \end{cases}$$

where $\Theta_0 = \{\theta_i;\ \theta \in (c,\infty)\}$, $\Theta = \mathbb{R}^1$ and the action a_0 accepts π_i as good and the action a_1 rejects π_i as not good. We are going to use a noninformative prior distribution $\tau(\theta_i, \sigma_i^2)$, where

$$\tau(\theta_i,\sigma_i^2) = \sigma_i^{-2} I_{(0,\infty)}(\sigma_i^2), \tag{3.2}$$

where $I(\cdot)$ is a usual indicator function. The preceding prior, in some sense, provides robustness.

Stage 1. Let $n_0 = \max\{2, [\frac{Z(1-\alpha)}{d}] + 1\}$, where 2d is the fixed width of the 100(1-2α)% HPD credible region which will be set up at Stage 2, and $Z_{(p)}$ is 100·p% (upper) percentile of the standard normal distribution. Let x_{ij} be a realization of X_{ij} and $\underset{\sim}{x}_i = (x_{i1},\ldots,x_{in_0})$ and $\bar{x}_i = \sum_{j=1}^{n_0} x_{ij}/n_0$. By definition, the marginal posterior distribution $\tau_1(\theta_i|\underset{\sim}{x}_i)$ of θ_i is a Student's t-distribution with (n_0-1) degrees of freedom, the location parameter $\bar{x}_i$, and the scale parameter $\sum_{j=1}^{n_0} (x_{ij}-\bar{x}_i)^2/n_0(n_0-1)$.
Then

$$E^{\tau_1(\theta_i|\underset{\sim}{x}_i)}(L(\theta_i,a_0)) = k_0 Pr(\Theta_1|\underset{\sim}{x}_i) \tag{3.3}$$

and

$$E^{\tau_1(\theta_i|\underset{\sim}{x}_i)}(L(\theta_i,a_1)) = k_1 Pr(\Theta_0|\underset{\sim}{x}_i). \tag{3.4}$$

Thus, at Stage 1, we retain π_i iff

$$k_0 Pr(\Theta_1|\underset{\sim}{x}_i) \le k_1 Pr(\Theta_0|\underset{\sim}{x}_i), \tag{3.5}$$

or, equivalently, we retain π_i iff

$$Pr(\Theta_1|\underset{\sim}{x}_i) \le \frac{k_1}{k_0+k_1}. \tag{3.6}$$

For an explicit explanation for $Pr(\Theta_1|\underset{\sim}{x}_i)$, see the following Lemma 1.

Lemma 1.

$$Pr(\Theta_1|\underset{\sim}{x}_i) = \int_{-\infty}^{c} dF^{\tau_1(\theta_i|\underset{\sim}{x}_i)}(\theta_i) = \begin{cases} 1 - \frac{1}{2} I_u(\frac{n_0-1}{2}, \frac{1}{2}) & \text{if } c-\bar{x}_i \ge 0, \\ \frac{1}{2} I_u(\frac{n_0-1}{2}, \frac{1}{2}) & \text{if } c-\bar{x}_i < 0, \end{cases}$$

where $I_x(a,b)$ is an incomplete Beta-function, $u = (n_0-1)/(n_0-1+t^2)$,

$t = (c-\bar{x}_i)/(s_i/\sqrt{n})$, and $s_i^2 = \sum_{j=1}^{n_0} (x_{ij}-\bar{x}_i)^2/(n_0-1)$.

Let S be the selected subset of Stage 1 and let s be its size. Then

(i) if s = 0, we decide that none of populations is good and stop,

(ii) if s = 1, we decide that the population selected is the only good one and the best at the same time and stop,

(iii) if $s \geq 2$, we proceed to Stage 2.

Stage 2. Now we want to set up a $100(1-2\alpha)\%$ HPD credible region $C_i(1-2\alpha)$ for θ_i of each population selected at Stage 1 with a common fixed width 2d.

A Procedure for constructing the credible region $C_i(1-2\alpha)$.

Let $g(\theta_i|n-1, \bar{x}_i, s_i^2/n)$ be the pdf of a Student's t-distribution with (n-1) degrees of freedom, the location parameter $\bar{x}_i$ and the scale parameter s_i^2/n, where $\bar{x}_i$ and s_i^2 are defined the same as before. Let $C_i(1-2\alpha) = (a_i, b_i)$. Then since $g(\theta_i|n-1, \bar{x}_i, s_i^2/n)$ is strongly unimodal and symmetric about $\bar{x}_i$, the following two equations provide the credible region $C_i(1-2\alpha)$.

(3.7) $$g(a_i|n-1, \bar{x}_i, s_i^2/n) = g(b_i|n-1, \bar{x}_i, s_i^2/n)$$

and

(3.8) $$\int_{a_i}^{b_i} g(\theta_i|n-1, \bar{x}_i, s_i^2/n)d\theta_i = 1-2\alpha.$$

Transform $\varepsilon_i = \sqrt{n}(\theta_i-\bar{x}_i)/s_i$ and by the equations (3.7) and (3.8),

(3.9) $$a_i + b_i = 2\bar{x}_i$$

and

$$(3.10)\qquad \Pr\{a_i < \theta_i < b_i\} = \Pr\{\frac{a_i-\bar{x}_i}{s_i/\sqrt{n}} < \xi_i < \frac{b_i-\bar{x}_i}{s_i/\sqrt{n}}\}$$

$$= \Pr\{|\xi_i| < \frac{\bar{x}_i-a_i}{s_i/\sqrt{n}}\}$$

$$= 1-I_u(\frac{n-1}{2}, \frac{1}{2})$$

$$= 1-2\alpha.$$

Then from the table of an incomplete Beta-function (e.g. Pearson (1934)), we can get the $100\cdot\alpha$ upper percentile point c_0 of the beta distribution. Hence by Lemma 1,

$$(3.11)\qquad u_{2\alpha} \equiv c_0 = \frac{n-1}{(n-1)+t^2}$$

$$= \frac{n-1}{(n-1) + \dfrac{(\bar{x}_i-a_i)^2}{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2/n(n-1)}}$$

$$= \frac{\sum_{j=1}^{n-1}(x_{ij}-\bar{x}_i)^2/n}{\sum_{j=1}^{n}\dfrac{(x_{ij}-\bar{x}_i)^2}{n} + (\bar{x}_i-a_i)^2}.$$

Thus

$$(3.12)\qquad a_i = \bar{x}_i - \sqrt{\frac{1}{c_0}-1}\sqrt{\frac{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2}{n}},$$

and

$$(3.13)\qquad b_i = \bar{x}_i + \sqrt{\frac{1}{c_0}-1}\sqrt{\frac{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2}{n}}.$$

Therefore the width 2d of the credible region $C_i(1-2\alpha)$ is

$$(3.14) \qquad 2d = 2\sqrt{\frac{1}{c_0}-1}\sqrt{\frac{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2}{n}}$$

and this implies that

$$(3.15) \qquad n = \frac{(\frac{1}{c_0}-1)\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2}{d^2}.$$

Hence a stopping rule N_i which provides a 100(1-2α)% HPD credible region $C_i(1-2\alpha)$ with a fixed width 2d is given by

$$(3.16) \qquad N_i = \inf\{n;\ n \geq n_0 \text{ and } n \geq [\frac{(\frac{1}{c_0}-1)\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2}{d^2}]+1\},$$

where [a] is the largest integer less than or equal to a. Note that after we stop sampling, the marginal posterior distribution $\tau_1(\theta_i|\underset{\sim}{x}_i)$ becomes a Students's t-distribution with (N_i-1) degrees of freedom, the location parameter $\bar{x}_i = \sum_{j=1}^{N_i} x_{ij}/N_i$ and the scale parameter $\sum_{j=1}^{N_i}(x_{ij}-\bar{x}_i)^2/N_i(N_i-1)$. At Stage 2, then we decide the population associated with the largest overall sample mean to be the best. That is, $\pi_{(s)}$, associated with $\bar{X}_{[s]}$, where $\bar{X}_{[1]} \leq \bar{X}_{[2]} \leq \cdots \leq \bar{X}_{[s]}$ are ordered means of usual sample means $\bar{X}_i$, is said to be the best among good ones.

<u>Lemma 2</u>. $\sqrt{n}\text{-}T\ (\frac{1}{c_0}-1)^{\frac{1}{2}} \to -Z_{(\alpha)} = Z_{(1-\alpha)}$ as $n \to \infty$.

Proof. The proof follows from the central limit theorem.

References

Alam, K. (1970). A two-sample procedure for selecting the population with the largest mean from k normal populations. Ann. Inst. Statist. Math. 22, 127-136.

Bechhofer, R. E. (1954). A single-sample multiple decision procedure for ranking means of normal populations with known variances. Ann. Math. Statist. 25, 16-39.

Bechhofer, R. E., Dunnett, C. W., and Sobel, M. (1954). A two-sample multiple decision procedure for ranking means of normal populations with a common unknown variance. Biometrika 41, 170-176.

Berger, J. O. (1980). Statistical Decision Theory: Foundations, Concepts, and Methods. Springer-Verlag, New York.

Bofinger, E. (1979). Two stage selection problem for normal populations with unequal variances. Aust. J. Statist., 21, 149-156.

Dudewicz, E. J., and Dalal, S. R. (1975). Allocation of observations in ranking and selection with unequal variances. Sankhyā Ser. B 37, 28-78.

Gupta, S. S. (1956). On a decision rule for a problem in ranking means. Ph.D. Thesis (Mimeo. Ser. No. 150). Inst. of Statist., University of North Carolina, Chapel Hill.

Gupta, S. S. and Kim, W. C. (1982). A two-stage elimination type procedure for selecting the largest of several normal means with a common unknown variance, Mimeo. Ser. No. 82-27, Dept. of Statist., Purdue University, W. Lafayette, IN.

Gupta, S. S. and Miescke, K. J. (1981). An essentially complete class of two-stage procedures with screening at the first stage. Mimeo. Ser. No. 81-53, Dept. of Statist., Purdue University, W. Lafayette, IN.

Gupta, S. S. and Miescke, K. J. (1982b). On the problem of finding a best population with respect to a control in two stages. Statistical Decision Theory and Related Topics III (S. S. Gupta and J. O. Berger eds.), Vol. 1, Academic Press, New York, 473-496.

Mukhopadhyay, N. (1979). Some comments on two-stage selection procedures. Commun. Statist. A8, 671-683.

Mukhopadhyay, N. (1980). A consistent and asymptotically efficient two-stage procedure to construct fixed width confidence intervals for the mean. Metrika, 27, 281-284.

Pearson, K. (ed) (1934). Tables of the Incomplete Beta Function, Cambridge University Press.

Rinott, Y. (1978). On two-stage procedures and related probability-inequalities. Commun. Statist.-Theor. Meth. A8, 799-811.

Tamhane, A. C. (1975). A minimax two-stage elimination type procedure for selecting the smallest normal variance. Tech. Report No. 260, Dept. of Operations Research, Cornell University, Ithaca, New York.

Tamhane, A. C., and Bechhofer, R. E. (1977). A two-stage minimax procedure with screening for selecting the largest normal mean. Commun. Statist.-Theor. Meth. A6, 1003-1033.

Tamhane, A. C. and Bechhofer, R. E. (1979). A two-stage minimax procedure with screening for selecting the largest normal mean (II): an improved PCS lower bound and associated tables. Commun. Statist. A8, 337-358.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We propose a two-stage procedure which select the best based on the so-called $100(1-2\alpha)\%$ Highest Posterior Density (HPD) credible regions with a common fixed-width 2d after retaining good populations based on the expected posterior losses. For an application, we use a noninformative prior distribution for the unknown means and variances of normal populations. Also we use 0-K_i losses at Stage 1 and a stopping rule N_i which provides a $100(1-2\alpha)\%$ HPD credible region with a fixed-width 2d to decide on the choice of the best population based on the overall sample means at Stage 2.

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